Math 107-Sec 450 **Exam 3** **Name:** ______________

Spring 2001 **Recitation Instructor:** ______________

No.	1	2	3	4	Total
score					

1. (18 pts) Consider the differential equation

$$\frac{dy}{dt} = -y^2(y-3). \qquad (1)$$

a.(4 pts) Find all equilibrium solutions for equation (1)

b.(6 pts) **Without solving** the differential equation, Sketch a reasonable graph of the solution $y(t)$ to equation (1) that satisfies $y(0) = 1$.

c.(8 pts) Use the Euler method with step size $\Delta x = 0.5$ to approximate the value of $y(1)$, where $y(t)$ is the solution to equation (1) with the initial condition $y(0) = 1$. **(Show your work, no calculator program is permitted)**

2. (21 pts, 7 pts for each part) Initially, a pool whose volume is 1000 liters is full with fresh water. Assume that water containing 5 grams of salt per liter is pumped into the pool at the rate of 100 liters per minute. The solution in the pool is kept well stirred and the excess water is drained out at the same rate (100 liters per minute). Let $y(t)$ denote the mass of the salt in the pool at time t.

   a. Write down a differential equation and an initial condition which $y(t)$ must satisfy.

   b. Solve (or write down the solution of) the initial value problem in part (a).

   c. At what time does the total amount of salt in the pool reach 3000 grams?

3. (13 pts) By using separation of variables, find an **explicit** formula for the solution to the IVP:

$$\frac{dy}{dx} = \frac{2xy}{x^2+1}, \qquad y(0) = 3.$$

4. (48 pts, 6 pts for each part) Given the points: $A = (-1, 1, 2)$, $B = (1, -1, 0)$, $C = (2, 1, 1)$.

a. Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

b. Find $\overrightarrow{AB} \cdot \overrightarrow{BC}$.

c. Find $\overrightarrow{AB} \times \overrightarrow{BC}$.

d. Does there exist a **unit** vector $\overrightarrow{u}$ that is orthogonal to $\overrightarrow{AB}$ and $\overrightarrow{u}$ is parallel to the vector $\langle -1, 1, 0 \rangle$? If so, find it, and if not show why.

e. Find the area of the triangle ABC.

f. Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

g. Find the equation of the plane containing the points A, B and C.

h. Find $Proj_{\overrightarrow{BC}} \overrightarrow{AB}$, the projection of $\overrightarrow{AB}$ onto $\overrightarrow{BC}$.